PP36789. Proposed by Michaly Bencze.

Prove that $2^{\sqrt{2}}+\left(\sqrt{6}\right)^{\sqrt{2}} \geq 6.$

Solution by Arkady Alt, San Jose, California, USA.

Since by AM-GM inequality $2^{\sqrt{2}}+\left(\sqrt{6}\right)^{\sqrt{2}} > 2\sqrt{2^{\sqrt{2}}\cdot\left(\sqrt{6}\right)^{\sqrt{2}}} = 2\cdot 2^{\frac{1}{\sqrt{2}}}\cdot 6^{\frac{1}{2\sqrt{2}}} =$ $2\cdot 2^{\frac{3}{2\sqrt{2}}}\cdot 3^{\frac{1}{2\sqrt{2}}}$ remains to prove inequality $2^{\frac{3}{2\sqrt{2}}}\cdot 3^{\frac{1}{2\sqrt{2}}} > 3 \Leftrightarrow 24 > 3^{2\sqrt{2}} \Leftrightarrow$ $8 > 3^{2\sqrt{2}-1}$. Since $2\sqrt{2}-1 < 2\cdot 1.42-1 = 1.84 < 1.85 = \dfrac{37}{20}$ we can to do one more reduction of original inequality, namely prove inequality $8 > 3^{37/20} \Leftrightarrow$ $2^{60} > 3^{37}$. We have $2^{8} = 256 > 243 > 3^{5}$ and $2^{52} > 3^{32} \Leftrightarrow 2^{13} > 3^{8} \Leftrightarrow$ $8192 > 6561$ and, therefore, $2^{60} > 3^{37}$.